

1 次の式を因数分解せよ。

$$\begin{aligned}
 (1) \quad & 2x^2 - 5xy - 3y^2 + 9x + 8y - 5 \\
 & = 2x^2 - (5y-9)x - (3y^2-8y+5) \\
 & = 2x^2 - (5y-9)x - (3y-5)(y-1) \\
 & = \{2x+(y-1)\}\{x-(3y-5)\} \\
 & = (2x+y-1)(x-3y+5)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & a^2(b-c) + b^2(c-a) + c^2(a-b) \\
 & = (b-c)a^2 - (b^2-c^2)a + (b^2c-bc^2) \\
 & = (b-c)a^2 - (b-c)(b+c)a + bc(b-c) \\
 & = (b-c)\{a^2 - (b+c)a + bc\} \\
 & = (b-c)(a-b)(a-c) \\
 & = -(a-b)(b-c)(c-a)
 \end{aligned}$$

2 次の式の 2 重根号をはずせ。

$$\begin{aligned}
 (1) \quad & \sqrt{8+2\sqrt{15}} \\
 & = \sqrt{5} + \sqrt{3} \\
 (2) \quad & \sqrt{6-4\sqrt{2}} \\
 & = \sqrt{6-2\sqrt{8}} \\
 & = 2-\sqrt{2} \\
 (3) \quad & \sqrt{4-\sqrt{15}} = \sqrt{\frac{8-2\sqrt{15}}{2}} = \frac{\sqrt{8-2\sqrt{15}}}{\sqrt{2}} \\
 & = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}} \\
 & = \frac{\sqrt{10}-\sqrt{6}}{2}
 \end{aligned}$$

3 次の問いに答えよ。

$$\begin{aligned}
 (1) \quad & (\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5}) \text{ を計算せよ。} \\
 & (\sqrt{2}+\sqrt{3})^2 - 5 = (5+2\sqrt{6}) - 5 = 2\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \text{ の分母を有理化せよ。} \\
 & \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-5} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(5+2\sqrt{6})-5} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}} \\
 & = \frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}
 \end{aligned}$$

4 $x = \frac{1}{\sqrt{7}+\sqrt{5}}, y = \frac{1}{\sqrt{7}-\sqrt{5}}$ のとき、次の式の値を求めよ。

$$\begin{aligned}
 (1) \quad & x+y \\
 & x = \frac{\sqrt{7}-\sqrt{5}}{2}, y = \frac{\sqrt{7}+\sqrt{5}}{2} \\
 & x+y = \sqrt{7} \\
 (2) \quad & xy \\
 & xy = \frac{1}{7-5} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & x^2 + y^2 \\
 & = (x+y)^2 - 2xy \\
 & = 7 - 1 \\
 & = 6
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & \frac{y}{x} + \frac{x}{y} \\
 & = \frac{y^2 + x^2}{xy} \\
 & = 6 \times 2 \\
 & = 12
 \end{aligned}$$

5 $\frac{1}{2-\sqrt{3}}$ の整数部分を a 、小数部分を b とする。

$$\begin{aligned}
 (1) \quad & a, b \text{ の値を求めよ。} \\
 & \frac{1}{2-\sqrt{3}} = 2+\sqrt{3} \text{ ㊦} \\
 & a=3, b=(2+\sqrt{3})-3 = \sqrt{3}-1
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & b + \frac{1}{b} \text{ の値を求めよ。} \\
 & (\sqrt{3}-1) + \frac{1}{\sqrt{3}-1} = (\sqrt{3}-1) + \frac{\sqrt{3}+1}{2} \\
 & = \frac{3\sqrt{3}-1}{2}
 \end{aligned}$$

6 方程式 $|x-6|=2x$ を解け。

$$\begin{aligned}
 & \cdot x \geq 6 \text{ ㊦} \\
 & x-6=2x \text{ ㊦, } x=-6 \text{ (不適)} \dots \text{①} \\
 & \cdot x < 6 \text{ ㊦} \\
 & -(x-6)=2x \text{ ㊦, } 3x=6 \\
 & \text{㊦, } x=2 \text{ (適当)} \dots \text{②} \\
 & \text{①, ② ㊦, } \underline{x=2}
 \end{aligned}$$

7 不等式 $|2x+5| < 8$ を満たす整数 x の個数を求めよ。

$$\begin{aligned}
 & -8 < 2x+5 < 8 \text{ ㊦} \\
 & -13 < 2x < 3, \quad -\frac{13}{2} < x < \frac{3}{2} \\
 & \text{㊦ 満たす整数 } x \text{ ㊦} \\
 & x = -6, -5, -4, -3, -2, -1, 0, 1 \text{ ㊦} \\
 & \underline{8 \text{ 個}}
 \end{aligned}$$

8 $\sqrt{(x+1)^2} - \sqrt{(x-2)^2}$ を次の各場合に分けて簡単にせよ。

(1) $x < -1$ の場合 $|x+1| - |x-2|$ より

$$-(x+1) + (x-2) = -3$$

(2) $-1 \leq x < 2$ の場合

$$(x+1) + (x-2) = 2x-1$$

(3) $x \geq 2$ の場合

$$(x+1) - (x-2) = 3$$

9 方程式 $|x| + |x-4| = 6$ を解け。

(Hint: $x < 0$, $0 \leq x < 4$, $x \geq 4$ の場合に分ける)

• $x < 0$ のとき

$$-x - (x-4) = 6 \quad \text{より} \quad 2x = -2$$

$$\text{よって} \quad x = -1 \quad (\text{適当}) \quad \dots \textcircled{1}$$

• $0 \leq x < 4$ のとき

$$x - (x-4) = 4 \quad \text{より} \quad \text{解なし} \quad \dots \textcircled{2}$$

• $x \geq 4$ のとき

$$x + (x-4) = 6 \quad \text{より} \quad 2x = 10$$

$$\text{よって} \quad x = 5 \quad (\text{適当}) \quad \dots \textcircled{3}$$

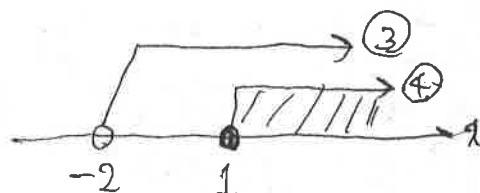
$$\textcircled{1} \sim \textcircled{3} \quad \text{より} \quad \underline{x = -1, 5}$$

10 次の連立不等式を解け。

$$\begin{cases} 6x+9 > 2x+1 & \dots \textcircled{1} \\ 3x-7 \leq 4(2x-3) & \dots \textcircled{2} \end{cases}$$

$$\textcircled{1} \text{ より} \quad 4x > -8 \quad \text{よって} \quad x > -2 \quad \dots \textcircled{3}$$

$$\textcircled{2} \text{ より} \quad 5x \geq 5 \quad \text{よって} \quad x \geq 1 \quad \dots \textcircled{4}$$



$$\textcircled{3}, \textcircled{4} \text{ より} \quad \underline{x \geq 1}$$